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Neutrino Oscillations: Status, Prospects and Opportunities at a Neutrino Factory

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Abstract

We review the current status of neutrino oscillations after 1258 days of Super-Kamiokande, assess their future prospects over the next 10 years as the next generation of experiments come on-line, and discuss the longer-term opportunities presented by a Neutrino Factory. We also give an introduction to the see-saw mechanism and its application to atmospheric and solar neutrinos.

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1 Introduction

The Super-Kamiokande experiment marks a watershed in neutrino oscillation physics. Essentially Super-Kamiokande is a large water Cherenkov detector consisting of 50,000 tons of pure water, and 11,200 photomultiplier tubes, making it the largest detector of its kind in the world. Neutrinos interact inside the detector producing either electrons or muons which give rise to the characteristic Cherenkov light cones which are observed by the photo-tubes situated on the walls of the detector. The results of the Super-Kamiokande experiment [1] provides compelling evidence for neutrino oscillations and hence neutrino mass, which if confirmed, would be the first evidence for new physics beyond the Standard Model.

The Super-Kamiokande experiment has measured the number of electron and muon neutrinos that arrive at the Earth's surface as a result of cosmic ray interactions in the upper atomosphere, which are referred to as "atmospheric neutrinos". While the number and and angular distribution of electron neutrinos is as expected, Super-Kamiokande has shown that the number of muon neutrinos is significantly smaller than expected and that the flux of muon neutrinos exhibits a strong dependence on the zenith angle. These observations give compelling evidence that muon neutrinos undergo flavour oscillations and this in turn implies that at least one neutrino flavour has a non-zero mass. The interpretation preferred by Super-Kamiokande is that muon neutrinos are oscillating into tau neutrinos. Several experiments that use neutrino beams from particle accelerators will take data over the next few years. If these experiments confirm the Super-Kamiokande results then this would be the first evidence that the Standard Model is incomplete since it was written down. This has led to an explosion activity in this area [2], [3].

Super-Kamiokande is also sensitive to the electron neutrinos arriving from the Sun, the "solar neutrinos", and has independently confirmed the reported deficit of such solar neutrinos long reported by other experiments. For example the Homestake Chlorine experiment which began data taking in 1970 consists of 615 tons of tetrachloroethylene, and uses radiochemical techniques to determine the Ar37 production rate. More recently the SAGE and Gallex experiments contain large amounts of Ga71 which is converted to Ge71 by low energy electron neutrinos arising from the dominant pp reaction in the Sun. The combined data from these and other experiments implies an energy dependent suppression of solar neutrinos which can be interpreted as due to flavour oscillations. Taken together with the atmospheric data, this requires that a second neutrino flavour has a non-zero mass. The minimal interpretation is that the electron neutrinos oscillate into a linear combination of muon and tau neutrinos.

There are a number of theories that can explain the experimental observations. Many of them are based on the see-saw mechanism [4, 5]. The see-saw mechanism assigns very heavy Majorana masses to the right-handed partners of the left-handed neutrinos, in addition to the usual Dirac masses which arise from the Yukawa couplings of left-handed neutrinos to right-handed neutrinos in the same way that charged lepton masses occur in the Standard Model. However the heavy Majorana masses of the right-handed neutrinos (which forbidden for the right-handed electron by electric charge conservation) leads to light physical Majorana neutrino masses via the see-saw mechanism. With a suitable choice of masses of the heavy Majorana masses, and Yukawa couplings, the masses and mixing angles of the light neutrino states fall into ranges which are suitable to accomodate the experimental observations.

Other theories explain the tiny neutrino mass in terms of Supersymmetry in which R-parity is violated, or in terms of extra dimensions. The R-parity explanations [6] offer the prospect of collider signatures which would confirm such explanations. The large extra dimension approach [7] predicts radical departures from the oscillation picture which could be tested by the next generation of neutrino experiments, and in addition this approach could be tested by collider experiments.

Whatever the underlying mechanism, a measurement of neutrino masses and mixing angles will allow us to infer information about the particular Grand Unified Theory (GUT) or string theory that governs the right-handed neutrinos. Furthermore, an accurate measurement of these parameters will provide information in the poorly understood lepton sector that will complement that already available in the quark sector. Such information will provide vital clues to the problem of the origin of quark and lepton masses and may unlock the puzzle of why there are three generations.

In section 2 we introduce neutrino masses and mixing angles. Section 3 describes the current status of neutrino oscillations while section 4 presents some simple patterns of neutrino masses which can account for atmospheric and solar oscillations. In section 5 we ask the question "What will we know in 10 years time?", and in section 6 we discuss the longer term opportunities for neutrino oscillation physics at a Neutrino Factory. Section 7 introduces the see-saw mechanism and shows how it may be applied to a hierarchical neutrino spectrum describing atmospheric and solar neutrino data using single right-handed neutrino dominance, and section 8 concludes the paper.

2 Neutrino Masses and Mixing Angles

The minimal neutrino sector required to account for the atmospheric and solar neutrino oscillation data consists of three light physical neutrinos with left-handed flavour eigenstates, ν_e, ν_μ, and ν_τ, defined to be those states that share the same electroweak doublet as the left-handed charged lepton mass eigenstates. Within the framework of three–neutrino oscillations, the neutrino flavor eigenstates ν_e, ν_μ, and ν_τ are related to the neutrino mass eigenstates ν_1, ν_2, and ν_3 with mass m_1, m_2, and m_3, respectively, by a 3×3 unitary matrix introduced by Maki, Nakagawa and Sakata (MNS) [8], U_{MNS},

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \tag{1}$$

Assuming the light neutrinos are Majorana, U_{MNS} can be parameterized in terms of three mixing angles θ_{ij} and three complex phases δ_{ij}. A unitary matrix has six phases but three of them are removed by the phase symmetry of the charged lepton Dirac masses. Since the neutrino masses are Majorana there is no additional phase symmetry associated with them, unlike the case of quark mixing where a further two phases may be removed. The MNS matrix may be expressed as a product of three complex Euler rotations,

$$U_{MNS} = U_{23}U_{13}U_{12} \tag{2}$$

where

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23}e^{-i\delta_{23}} \\ 0 & -s_{23}e^{i\delta_{23}} & c_{23} \end{pmatrix} \tag{3}$$

$$U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \tag{4}$$

$$U_{12} = \begin{pmatrix} c_{12} & s_{12}e^{-i\delta_{12}} & 0 \\ -s_{12}e^{i\delta_{12}} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Note that the allowed range of the angles is $0 \le \theta_{ij} \le \pi/2$. Since we have assumed that the neutrinos are Majorana, there are two extra phases, but only one combination $\delta = \delta_{13} - \delta_{23} - \delta_{12}$ affects oscillations. For the purposes of studying oscillation physics we may take $\delta = \delta_{13}$, and $\delta_{23} = \delta_{12} = 0$, so that U_{MNS} resembles the CKM matrix,

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{6}$$

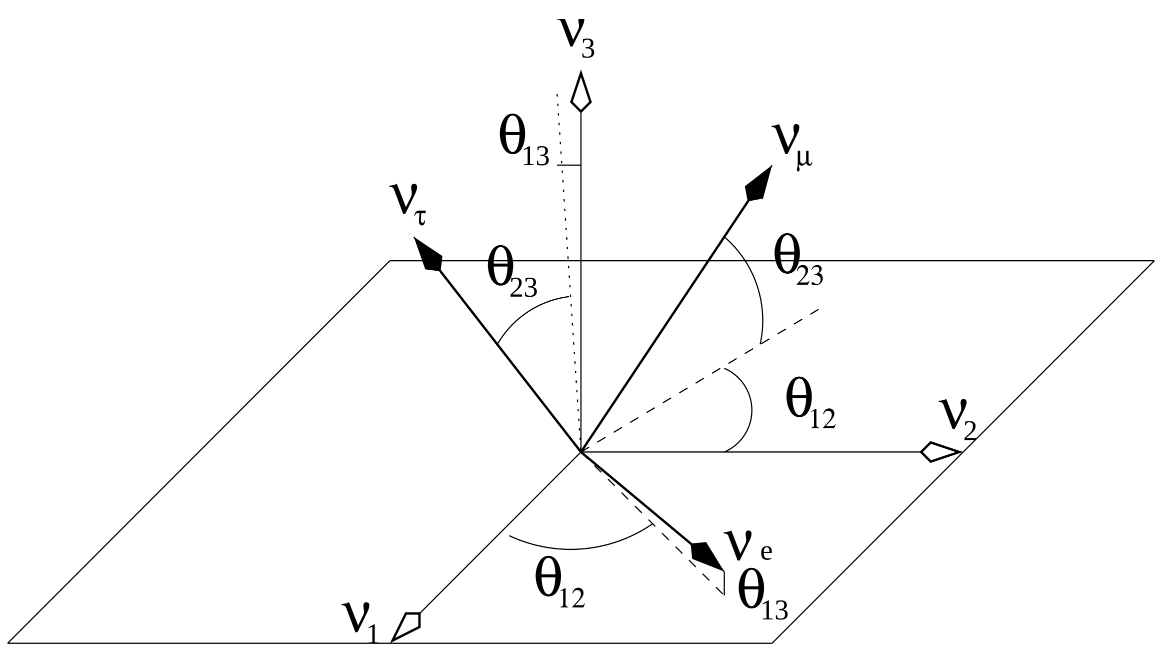

Figure 1: The relation between the neutrino flavor eigenstates ν_e, ν_μ, and ν_τ and the neutrino mass eigenstates ν_1, ν_2, and ν_3 in terms of the three mixing angles θ_{12}, θ_{13}, θ_{23}.

It is convenient to define:

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \ . \tag{7}$$

Oscillation probabilities depend upon the time–of–flight (and hence the baseline L), the Δm_{ij}^2, and U_{MNS} (and hence $\theta_{12}, \theta_{23}, \theta_{13}$, and δ).

Ignoring phases, the relation between the neutrino flavor eigenstates ν_e, ν_μ, and ν_τ and the neutrino mass eigenstates ν_1, ν_2, and ν_3 is just given as a product of three Euler rotations as depicted in Figure 1.

3 Status of Neutrino Oscillations

Current atmospheric neutrino oscillation data are well described by simple two-state mixing

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{23} & s_{23} \\ -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix} \ , \tag{8}$$

and the two-state Probability oscillation formula

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta_{23} \sin^2(1.27 \Delta m_{32}^2 L/E) \tag{9}$$

where Δm_{32}^2 is in units of eV^2, the baseline L is in km and the beam energy E is in GeV. The atmospheric data is statistically dominated by the Super-Kamiokande results and the latest 1117 day data sample leads to [2]:

- $\sin^2 2\theta_{23} > 0.88$
- $1.5 \times 10^{-3}\ \text{eV}^2 < |\Delta m_{32}^2| < 5 \times 10^{-3}\ \text{eV}^2$ (90% CL)

CHOOZ data from $\bar{\nu}_e \to \bar{\nu}_e$ disappearance not being observed [9] provides a significant constraint on θ_{13} over the Super-Kamiokande (SK) prefered range of Δm_{32}^2:

- $\sin^2 2\theta_{13} < 0.1 - 0.3$

The solar neutrino problem can be solved by four different combinations of oscillation parameters, three of which are based on the MSW mechanism [10], namely the large mixing angle (LMA), the small mixing angle (SMA) and the long wavelength (LOW) solutions, and a fourth solution represents vacuum (VAC) oscillations [11]. Various groups have performed fits to the solar and atmosheric solutions [13, 14, 15]. Representative values of the

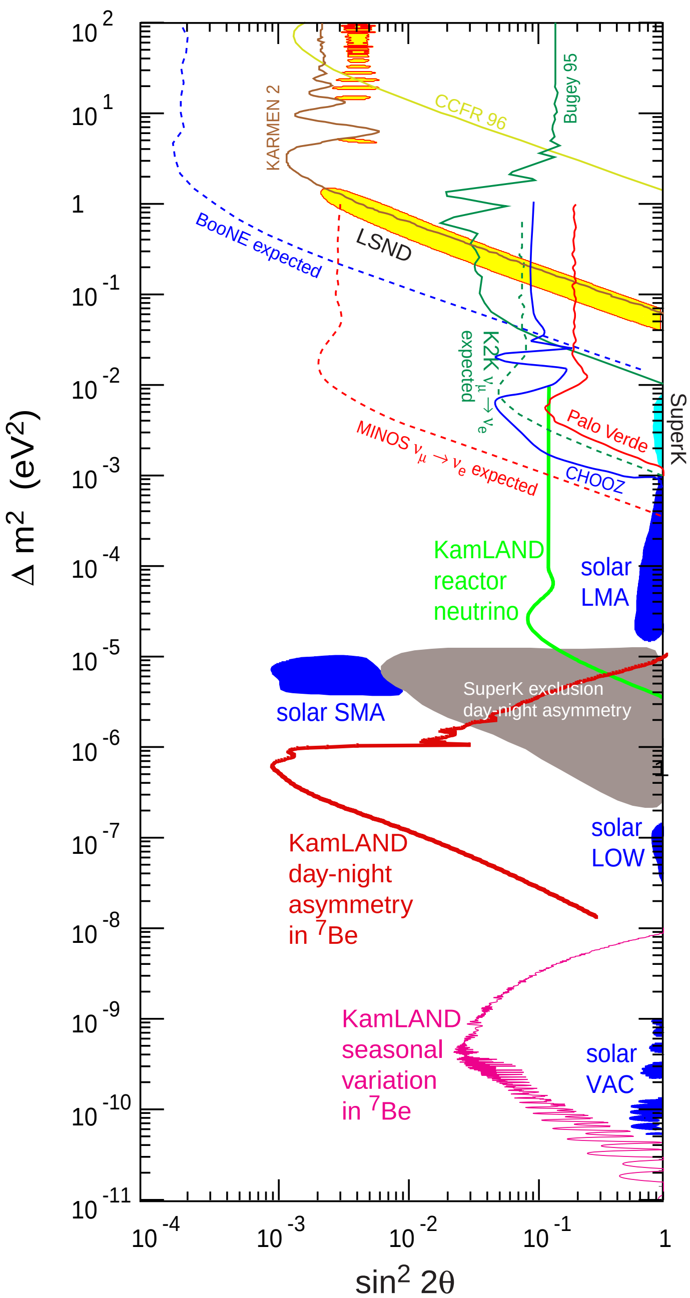

Figure 2: Summary of all oscillation data (taken from [17]).

	$\sin^2 2\theta_{12}$	$\Delta m^2_{21}\quad (eV^2)$
LMA	0.78	3.3×10^{-5}
SMA	0.0027	5.1×10^{-6}
LOW	0.93	9.6×10^{-8}
VAC	0.93	8×10^{-10}

Table 1: Typical best fit solar solutions (from [12]).

parameters associated with the four solutions based on the most recent data are given in Table 1 [12].

In Figure 2 we give a simplified summary of all oscillation data, based on two-flavour analyses, and ignoring the "dark side" of the parameter space corresponding to assuming that $\theta_{ij} \leq \pi/4$ [16]. For large angle mixing which depends on matter enhancement effects it becomes a relevant question whether $\theta_{ij} < \pi/4$ or $\theta_{ij} > \pi/4$, but the VAC solution is symmetrical about $\theta_{12} = \pi/4$. This figure, taken from [17] also includes a signal of neutrino mass from the LSND experiment [18]. If confirmed this would require the introduction of a fourth physical neutrino flavour, in order to achieve three different mass squared splittings. Since LEP data from the Z boson width only allows three light neutrinos, the fourth light neutrino would have to be a sterile neutrino which does not carry any quantum numbers, unlike the three sequential neutrinos which carry weak quantum numbers. We shall not consider the implications of the LSND result any further here.

Recently Super-Kamiokande have reported constraints on neutrino oscillations using 1258 days of their solar neutrino data [19]. They do not see any significant energy spectrum distortion, or day/night effect, leading to strong constraints on the allowed regions as shown in Figure 3. They claim that for active neutrino oscillations, the SMA and VAC solutions are disfavoured at 97% C.L. from the (absence of the) day/night effect, and sterile neutrino oscillations are disfavoured at 95% C.L. They conclude that LMA active neutrino oscillations are preferred. The results in Table 1 [12] are also based on an analysis performed on a similar data set, but using a different global analysis technique, and concludes that all four solar solutions remain viable although LMA is slightly preferred. Part of the confusion is due to the fact that it is the absence of any "smoking guns" for neutrino oscillations which is leading to the constraints. Interestingly if only the Super-Kamiokande data is considered, and the other solar neutrino experiments are removed from the analysis, then a large region of the LOW solution opens up for both active and sterile neutrinos since there would be no "smoking gun" sensitivity from either the day/night effect or the spectral distortion [19] in this region.

To summarise the current experimental situation, it is probably premature to exclude any of the four solar solutions in Table 1. We can be rea-

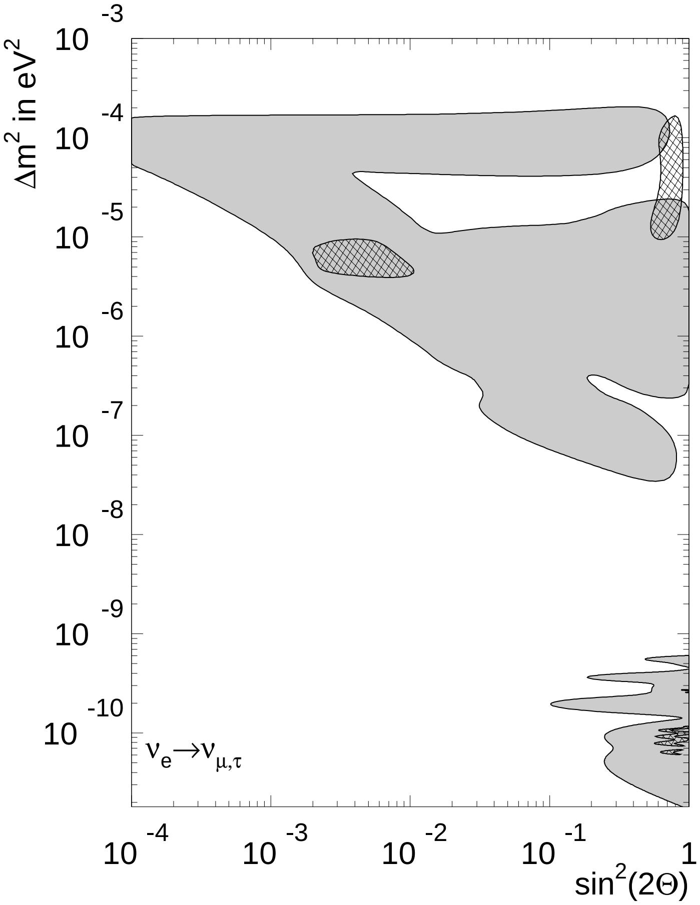

Figure 3: The Super-Kamiokande day-night exclusion area (shaded) based on 1258 days of data compared to the flux allowed area (hatched) from all experiments [19].

sonably confident that $\theta_{23} \approx \pi/4$, $\theta_{13} \leq 0.1$, while θ_{12} might be either large or small. Although nothing is known about the absolute scale or ordering of the mass squareds m_i^2, we do know that the ratio

$$R \equiv |\Delta m^2_{21}|/|\Delta m^2_{32}| \equiv |\Delta m^2_{sol}|/|\Delta m^2_{atm}| \tag{10}$$

shows a hierarchy which, however, depending on the solar solution could be rather mild, or relatively strong:

- $R = \mathcal{O}(10^{-2})$ (LMA)
- $R = \mathcal{O}(10^{-7})$ (VAC)

4 Patterns of Neutrino Masses

There are two simple patterns of neutrino masses which can account for the atmospheric and solar neutrino data, as shown in Figure 4. These are based on either the hierarchy $|m_3| \gg |m_2| \gg |m_1|$ (scheme A) or the inverted hierarchy $|m_1| \approx |m_2| \gg |m_3|$ (scheme B). In the case of a three neutrino hierarchical spectrum (scheme A), there will be one physical neutrino with a mass of about $|m_3| \approx 5 \times 10^{-2}$ eV, which would contribute as much "hot" dark matter to the universe as all the visible stars, and so would be of some relevance for cosmological structure formation. A second physical neutrino with a mass of about $|m_2| \approx 5 \times 10^{-3}$ eV must also exist in order to account for the solar oscillation data. In the case of an inverted hierarchy [21] (scheme B) there are two neutrinos each with a mass of about $|m_1| \approx |m_2| \approx 5 \times 10^{-2}$ eV but with a small mass splitting of order 2×10^{-4} eV, giving twice as much "hot" dark matter. Therefore knowing the neutrino spectrum would be of some interest for cosmological dark matter even in these minimal scenarios, although the amount of dark matter in both cases is very small. The inverted spectrum is technically natural, in the sense of being stable under radiative corrections, and predicts a large solar angle [21].

In addition a third pattern of neutrino masses has been proposed (scheme C) based on all three neutrinos being approximately degenerate with a mass of order an eV, but with very small mass splittings suitable to describe the atmospheric and solar data. In this approximately degenerate case there could be a substantial component of "hot" dark matter, although in general such a pattern of neutrino masses tends to be unstable under radiative corrections unless protected by a symmetry [22].

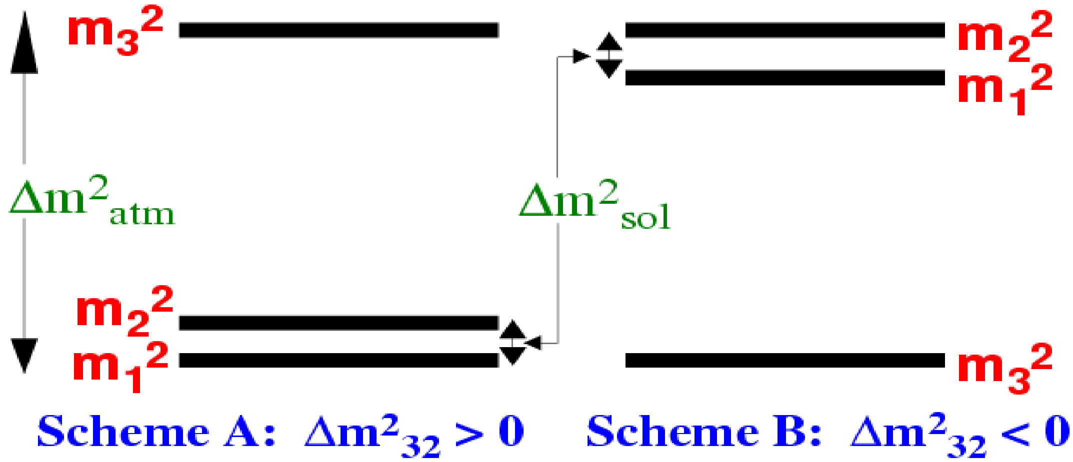

Figure 4: Alternative neutrino mass patterns that are consistent with neutrino oscillation explanations of the atmospheric and solar neutrino deficits (taken from ref.[20].)

5 What Will We Know in 10 years Time?

The answer to this question is not completely clear, which is one of the main reasons why neutrino physics is so exciting at the current time. In the near future much better solar neutrino measurements will be available as SNO, KamLAND, and Borexino furnish us with new data [23, 24, 25]. For example, Borexino is a new solar neutrino experiment at Gran Sasso which will measure the flux of the low energy Be7 neutrinos in real time, which is expected to be suppressed according to the MSW solutions. Taken together these experiments will provide "smoking gun" evidence for neutrino oscillations, if it exists, in all the currently allowed solar parameter ranges.

The Sudbury Neutrino Observatory (SNO) experiment is presently taking data and the first results are expected very soon. The SNO detector contains 1000 tonnes of heavy water D2O and 9600 photo tubes. The use of heavy water, especially when enriched with salt, will enable the neutral current (Z exchange) interactions of the neutrinos to be measured, and hence the total active neutrino flux to be measured. Thus SNO will see an excess of neutral current events for flavour oscillations irrespective of the values taken by the mixing parameters. A combination of spectral distortions, day-night rate differences, and seasonal rate differences in the above experiments will then be used to distinguish betwen the currently allowed parameter ranges.

KamLAND (Kamioka Liquid scintillator Anti-Neutrino Detector) will study antineutrinos produced by nearby nuclear reactors, and will for example cover the LMA MSW region "in the laboratory". Of particular interest to the Neutrino Factory proposal is the ability of KamLAND to confirm neutrino oscillations with a terrestrial experiment if the parameters lie in the LMA range necessary for CP violation to be observed at a Neutrino Factory (as discussed later). It is difficult to say how accurately θ_{12} and Δm^2_{21} will be measured as this depends on the values of the other parameters, however an accuracy of $\sim 10-20\%$ seems possible. It is also possible thst within 10 years we would have a real-time measurement of the pp solar neutrino spectrum from SIREN, LENS or HELLAZ [26, 27, 28]. This would improve the accuracy of our parameter determination.

Over the next ten years the long baseline (LBL) experiments such as K2K, MINOS and eventually the CERN to Gran Sasso experiments will report new results [29, 30, 31]. The K2K experiment (a neutrino beam from from KEK to Super-Kamiokande) has already reported results from its first two years of running [29]. These experiments are capable of confirming or rejecting the hypothesis that neutrino oscillations are required to explain Super-Kamiokande atmospheric neutrino results. If the Super-Kamiokande results are confirmed then θ_{23} and Δm^2_{32} may be measured with an accuracy of about $\sim 10\%$. MINOS will be sensitive to $\sin^2 2\theta_{13}$ values as low as 0.01 [30]. In addition to these LBL experiments there is a proposal to build a "super" neutrino beam line at the recently approved Japanese Hadron Facility (JHF) [32]. This beam would illuminate the Super-Kamiokande detector, and enable a more accurate determination of $\sin^2 2\theta_{13}$. Discussions have also begun [33] on building such a high-flux beamline at Fermilab. In addition, it is proposed to use the proton driver for the CERN Neutrino Factory, the SPL, to provide a low energy neutrino beam, pointing at the Modane laboratory in the Alps. This will form the first stage of the CERN Neutrino Factory project. However, the precision of these experiments will be limited by the uncertainty in the beam flavour content. For example, K2K currently quote an error of 7%, while MINOS hopes to reduce this to 2%. Such super-beam projects may be regarded as a natural stepping stone to a Neutrino Factory, to which we now turn.

6 Opportunities at a Neutrino Factory

What is a Neutrino Factory? Basically it is a high intensity muon storage ring with long straight sections along which the muons decay to deliver a high intensity beam of neutrinos [34, 35, 36]. A typical high performance Neutrino Factory would involve 50 GeV muon beams and would deliver about 10^{20} muon decays per year. The resulting neutrino beams are clearly of very high energy and intensity and have precisely predictable neutrino flavour content, making them superior to the conventional neutrino beams such as those proposed for example in [32, 33].

Despite the anticipated progress, even in 10 years time there are three quantities which will remain obscure even in the most optimistic scenarios, and these quantities would be measurable at a Neutrino Factory. We shall refer to them as the three missing si(g)ns:

1. The CHOOZ angle $\sin^2 2\theta_{13}$ which will still be ill-determined even after LBL experiments, and is measureable down to an astonishing 0.0001 at a Neutrino Factory.

2. The CP violating phase $\sin\delta$ which is impossible to measure unless we have the LMA MSW solution.

3. The sign of the 23 mass splitting (Δm^2_{32}), which remains ambiguous as shown in Figure 4, is easy to measure at a Neutrino Factory.

It is worth emphasising that an accurate measurement of $\sin^2 2\theta_{13}$ will be important in discriminating GUT and string theories [37], and that it if this angle is smaller than 0.01 it will simply be unmeasured by the LBL experiments. At a Neutrino Factory it is relatively straightforward to measure this angle using the Golden Signature of "wrong sign" muons. For example suppose there are positive muons circulating in the storage ring, then these decay as $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ giving a mixed beam of electron neutrinos and muon anti-neutrinos. The muon anti-neutrinos will interact in the far detector to produce positive muons. Any "wrong sign" negative muons which may be observed can only arise from the neutrino oscillation of electron neutrinos into muon neutrinos with probability given by a CP conserving part P^+ and a a CP violating part P^-. The exact formulae in vacuum are given by:

$$P(\nu_e \to \nu_\mu) = P^+(\nu_e \to \nu_\mu) + P^-(\nu_e \to \nu_\mu) \tag{11}$$

where the CP conserving part is

$$\begin{aligned} P^+(\nu_e \to \nu_\mu) = & - 4Re(U_{e1}U^*_{\mu1}U^*_{e2}U_{\mu2})\sin^2(1.27\Delta m^2_{21}L/E) \\ & - 4Re(U_{e1}U^*_{\mu1}U^*_{e3}U_{\mu3})\sin^2(1.27\Delta m^2_{31}L/E) \\ & - 4Re(U_{e2}U^*_{\mu2}U^*_{e3}U_{\mu3})\sin^2(1.27\Delta m^2_{32}L/E) \end{aligned} \tag{12}$$

and the CP violating part is

$$\begin{aligned} & P^-(\nu_e \to \nu_\mu) = -c_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta \\ \times \quad & \sin(1.27\Delta m^2_{21}L/E)\sin(1.27\Delta m^2_{31}L/E)\sin(1.27\Delta m^2_{32}L/E) \end{aligned} \tag{13}$$

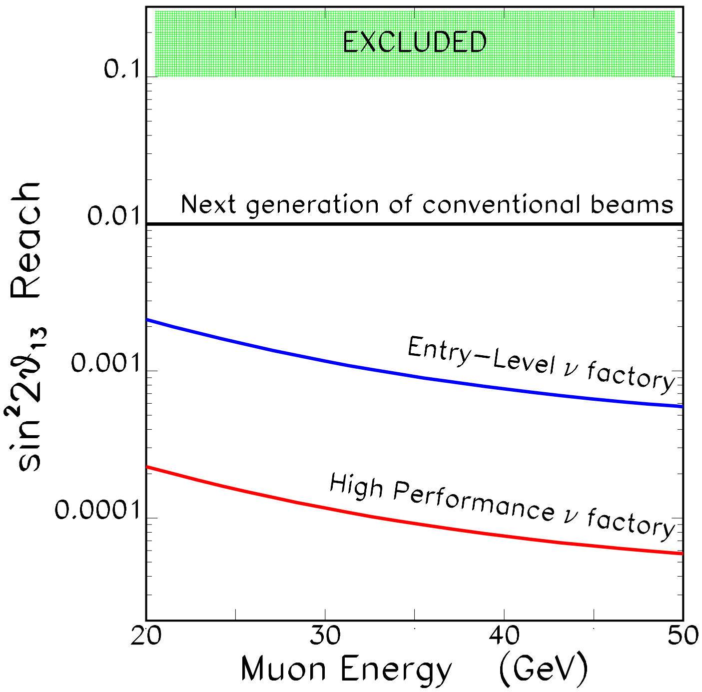

Figure 5: Reach in θ_{13} at a Neutrino Factory (taken from ref.[20].)

Note that P^- requires all three families to contribute, and it vanishes if any mixing angle or mass splitting is zero. The angle θ_{13} may easily be extracted from U_{e3} in the dominant CP conserving term P^+, leading to the expected limits in this angle at a Neutrino Factory as shown in Figure 5.

In order to determine the CP violating phase $\sin\delta$ it is necessary to measure the CP violating term P^-. In order to do this one must compare the result for $P(\nu_e \to \nu_\mu)$ to the result to the case where the positive muons in the storage ring are replaced by negative muons and the analagous experiment is performed to measure $P(\bar{\nu}_e \to \bar{\nu}_\mu)$. The CP violating asymmetry due to the CP violating phase δ is given by

$$A^\delta = \frac{P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\mu)}{P(\nu_e \to \nu_\mu) + P(\bar{\nu}_e \to \bar{\nu}_\mu)} \tag{14}$$

from which we obtain

$$A^{\delta} = \frac{P^{-}(\nu_e \to \nu_\mu)}{P^{+}(\nu_e \to \nu_\mu)} \approx \frac{\sin 2\theta_{12} \sin \delta}{\sin \theta_{13}} \sin(1.27 \Delta m_{21}^2 L/E) \tag{15}$$

It is clear that in order to measure the CP asymmetry we require large θ_{12} and large Δm_{21}^2 and this corresponds to the LMA MSW solution. In addition we require large $\sin \delta$. Also it would seem that having small θ_{13} enhances the CP asymmetry, however it should be remembered that the CP asymmetric rate P^- in Eq.13 is proportional to $\sin 2\theta_{13}$, and so θ_{13} should not be too small otherwise the number of events will be too small.

Unfortunately life is not quite as simple as the above discussion portrays. The Earth is made from matter and not anti-matter and so CP will be violated by matter effects as the neutrino beam passes through the Earth from the muon storage ring to the far detector. For example the matter effects will modify the formulas for $P(\nu_e \to \nu_\mu)$ involving θ_{13} and Δm_{31}^2 as follows:

$$\begin{aligned} \sin 2\theta_{13} &\to \frac{\sin 2\theta_{13}}{\left(\frac{A}{\Delta m_{31}^2} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}} \\ \Delta m_{31}^2 &\to \Delta m_{31}^2 \sqrt{\left(\frac{A}{\Delta m_{31}^2} - \cos 2\theta_{13}\right)^2 + \sin^2 2\theta_{13}} \end{aligned} \tag{16}$$

where

$$A = 7.6 \times 10^{-5} \rho E \tag{17}$$

where ρ is the density of the Earth in gcm^{-3} and E is the beam energy in GeV. The point is that for $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ the sign of A is reversed. From one point of view this is good news, since unlike the vacuum oscillation formulae, Δm_{31}^2 enters linearly, not quadratically, and so matter effects enable the sign of the mass squared splitting to be determined in a rather straightforward way, as shown in Figure 6.

However from the point of view of measuring $\sin \delta$ it leads to complications since the asymmetry in the rate in Eq.14 can get contributions from both intrinsic CP violation and from matter induced CP violation, and the measured asymmetry is a sum of the two effects

$$A^{CP} = A^{\delta} + A^{matter} \tag{18}$$

Since both effects are by themselves rather small, it will be a very difficult job to disentangle them, and the optimal strategy continues to be studied [34, 35, 36, 38]. The optimal place to sit in order to observe CP violation seems

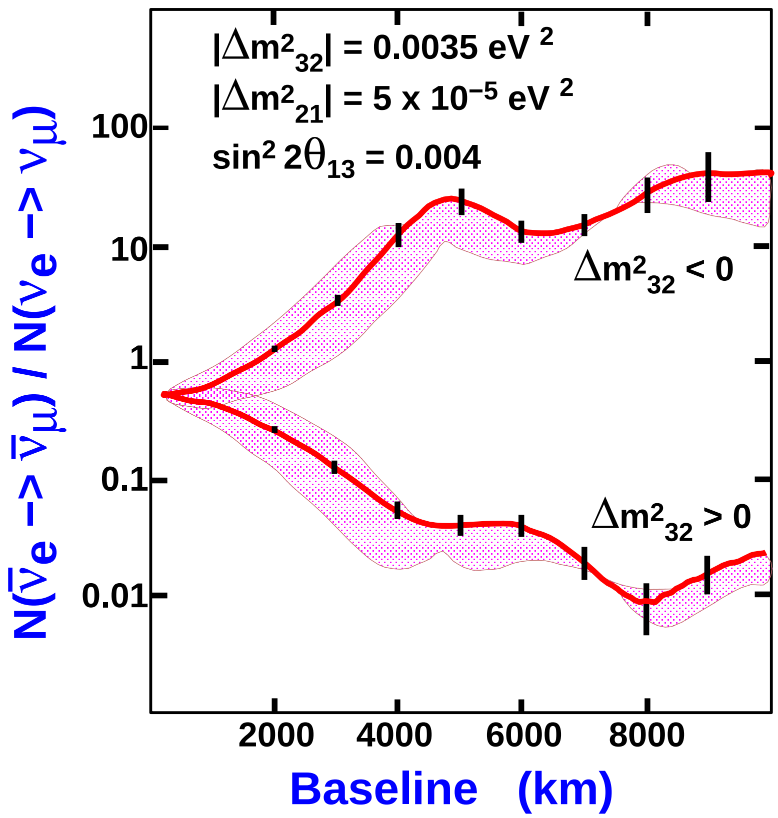

Figure 6: Measuring δ and sign(Δm^2_{32}) at a Neutrino Factory. In this plot the bands correspond to varying δ between 0 and $\pi/2$, and the error bars are estimated for a high performance Neutrino Factory (taken from ref.[20].)

to be at the peak of $\sin(1.27\Delta m^2_{32}L/E)$ in order to maximise P^- according to Eq.13 (certainly we should avoid being at its node otherwise CP violation vanishes). In order to do this efficiently it may be desirable to have energy-tunable beams, and it is certainly necessary to have a good understanding of the density profile of the Earth. Assuming the LMA solution, the prospects for measuring CP violation at a Neutrino Factory are good as shown in Figure 6.

7 More About the See-Saw Mechanism

The basic idea of the see-saw mechanism [4] can most simply be explained in the context of a single family of neutrinos, where the Dirac mass coupling the left-handed neutrino to the right-handed neutrino is m_{LR} and the Majorana mass of the right-handed neutrino is M_{RR}. The neutrino masses in the basis of left and right-handed neutrinos can then be written as a mass matrix

$$\begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \tag{19}$$

where in this one family example $m_{LR}^T = m_{LR}$. If $m_{LR} \ll M_{RR}$ then when this mass matrix is diagonalised we find that one of the eigenstates may be identified to good approximation with the left-handed neutrino and has a very small Majorana mass $m_{LL} \approx m_{LR} M_{RR}^{-1} m_{LR}^T$. For example if we take $m_{LR} = M_W$ and $M_{RR} = M_{GUT}$ then we find $m_{LL} \sim 10^{-3}$ eV which looks good for solar neutrinos. Atmospheric neutrino masses would require a right-handed neutrino with a mass below the GUT scale.

Generalising the above result to three families of neutrinos the light effective physical left-handed neutrino Majorana masses are now given by a three by three matrix m_{LL} which is given by $m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^T$ where m_{LR} is the Dirac neutrino mass matrix, typically of the same magnitude as the charged lepton mass matrix, and M_{RR} is a heavy right-handed neutrino Majorana matrix whose entries may be as large as the GUT scale. The eigenvalues of m_{LL} are the physical neutrino masses m_1, m_2, and m_3, and, in the diagonal charged lepton mass basis, the matrix which diagonalises m_{LL} is the neutrino mixing matrix U_{MNS}.

Consider three left-handed neutrinos but only two [2] right-handed neutrinos. Then let us write the Dirac mass matrix as

$$m_{LR} = \begin{pmatrix} 0 & a & d \\ 0 & b & e \\ 0 & c & f \end{pmatrix} \tag{20}$$

where the notation LR means that the second and third columns of m_{LR} correspond to the second and third right-handed neutrinos. The heavy Majorana mass matrix, assumed to be diagonal, is

$$M_{RR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Y \end{pmatrix} \tag{21}$$

[2] This is purely for simplicity. The following argument also applies to three right-handed neutrinos but is more cumbersome. Note that with three left-handed neutrinos m_{LL} is always a three by three matrix regardless of the number of right-handed neutrinos.

Then using the see-saw formula for the light effective Majorana mass matrix $m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^T$, we find the symmetric matrix,

$$m_{LL} = \begin{pmatrix} \frac{d^2}{Y} + \frac{a^2}{X} & \frac{de}{Y} + \frac{ab}{X} & \frac{df}{Y} + \frac{ac}{X} \\ . & \frac{e^2}{Y} + \frac{b^2}{X} & \frac{ef}{Y} + \frac{bc}{X} \\ . & . & \frac{f^2}{Y} + \frac{c^2}{X} \end{pmatrix} \tag{22}$$

So far the discussion is fairly general, although we have assumed a diagonal heavy Majorana mass matrix. In order to account for the atmospheric and solar neutrino data many models have been proposed based on the see-saw mechanism [5]. One question which is common to all these models is how to arrange for a large mixing angle involving the second and third generation of neutrinos, without destroying the hierarchy of mass splittings in Eq.10, $R \equiv |\Delta m^2_{sol}|/|\Delta m^2_{atm}|$. Assuming $\theta_{23} \sim \pi/4$ one might expect two similar eigenvalues $m_2 \sim m_3$, and then the hierarchy of scheme A in Figure 4 seems rather unnatural.

One way to achieve a natural hierarchy is to suppose that the third right-handed neutrino contributions are much greater than the second right-handed neutrino contributions in the 23 block of m_{LL} [39],

$$\frac{(e^2, f^2, ef)}{Y} \gg \frac{(b^2, c^2, bc)}{X} \tag{23}$$

This implies an approximately vanishing 23 subdeterminant,

$$det[m_{LL}]_{23} = \left(\frac{e^2}{Y} + \frac{b^2}{X}\right)\left(\frac{f^2}{Y} + \frac{c^2}{X}\right) - \left(\frac{ef}{Y} + \frac{bc}{X}\right)^2 \approx 0 \tag{24}$$

The 23 subdeterminant is also equal to the product of the eigenvalues

$$det[m_{LL}]_{23} = m_2 m_3 \tag{25}$$

and hence from Eqs.24,25

$$m_2/m_3 \ll 1 \tag{26}$$

Thus the assumption in Eq.23 that the third right-handed neutrino gives the dominant contribution to the 23 block of m_{LL} naturally leads to a neutrino mass hierarchy. This mechanism is called single right-handed neutrino dominance (SRHND)[39]. In the limit that only a single right handed neutrino contributes the determinant clearly exactly vanishes and we have $m_2 = 0$ exactly. However the sub-dominant contributions from the second right-handed neutrino will give a small finite mass $m_2 \neq 0$ as required by the MSW solution to the solar neutrino problem.

Developing the simple example above a little further, assuming SRHND as discussed above, we may obtain simple analytic estimates for the neutrino masses:

$$m_1 = 0, \tag{27}$$

$$m_2 \sim \frac{(b-c)^2}{X} \tag{28}$$

$$m_3 \sim \frac{(d^2+e^2+f^2)}{Y} \tag{29}$$

Note that m_3 (m_2) is determined by parameters associated with the dominant (subdominant) right-handed neutrino. Given the SRHND assumption in Eq.23 we see that we have a generated a hierarchical spectrum $m_1 \ll m_2 \ll m_3$ as in scheme A of Figure 4.

The mixing angles may also be estimated to be [39]

$$\tan\theta_{23} \sim \frac{e}{f}, \tag{30}$$

$$\tan\theta_{13} \sim \frac{d}{\sqrt{e^2+f^2}} \tag{31}$$

$$\tan\theta_{12} \sim \frac{a}{b-c} \tag{32}$$

For example by a suitable choice of parameters $e = f \gg d$ and $a \sim (b-c)$ it is possible to have large θ_{12} suitable for the LMA or LOW solution and a maximal θ_{23} suitable for atmospheric oscillations, while maintaining a small θ_{13} consistent with the CHOOZ constraint. Note that the hierarchical masses in Eqs.27,28,29 are controlled by the SRHND condition Eq.23 and the hierarchy is unaffected by the large angle conditions above. This is due to the approximately vanishing 23 subdeterminant of m_{LL}, and the underlying physical mechanism responsible is SRHND.

These ideas may be implemented in flavour models based on a $U(1)$ family symmetry but the predictions for different models are uncertain due to unknown coefficients multiplying expansion parameters in the neutrino matrices. Within such a framework the best one can do is scan over the unknown coefficients, and plot distributions in the predicted quantities [37]. See-saw models with SRHND are clearly seen to give distributions in R (defined in Eq.10) which peak at smaller values than other models, so the idea of SRHND is testable by a future measurement of R. The result of such scans shows that even without the SRHND mechanism it is possible to achieve mild hierarchies corresponding to say $R \sim 0.1$ in a perfectly natural way. However if experiment tells us that $R \leq 10^{-2}$ then an accidental hierarchy of this magnitude looks increasingly unlikely, and in this case the simplest interpretation might be SRHND.

8 Conclusion

Despite the impressive progress we still in reality know very little about the neutrino spectrum. For example, the pattern of masses could be hierarchical, inverted or degenerate. According to the recent Super-Kamiokande results which favour the LMA solution with active neutrinos, the most likely scenario is approximate bimaximal mixing $\theta_{23} \approx \pi/4$, $\theta_{12} \approx \pi/4$, $\theta_{13} \approx 0$ [40],

$$U_{MNS} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{33}$$

However this interpretation is far from certain at this stage. The solar angle might still be very small, as in the SMA solution. There could still be a fourth (sterile) neutrino state. One or more of the solar experiments could be wrong; for example if the Homestake Chlorine data is removed then the allowed parameter space opens up significantly, and the distinction between the LMA, LOW and VAC solutions becomes less pronounced. Then even trimaximal mixing with a large CHOOZ angle cannot strictly be excluded [41]. Finally perhaps we are not seeing oscillations, but some energy independent suppression for example due to the effects of large extra dimensions, or perhaps some other non-standard effect. Fortunately the answers to these questions will be found by the forthcoming generation of neutrino experiments. As Super-Kamiokande leaves centre stage, other experiments such as those discussed in section 5 are set to enter the spotlight. The new generation of solar neutrino experiments will be able to confirm solar neutrino oscillations, and the LBL experiments will be able to confirm atmospheric oscillations. Together they will either confound the sceptics, or else dumbfound the believers, and it is a brave person who would predict that there will be no more surprises in store.

For example at the time of writing we are awaiting the first results from the first year of operation of SNO with great anticipation. The first year of data will consist of a measurement of the charged current scattering rate from deuterium, which will provide a clean measurement of the electron neutrino flux. By contrast Super-Kamiokande measures the elastic scattering rate from electrons, which includes charged and neutral current contributions. By comparing the charged current suppression measured by SNO to the elastic scattering suppression measured by Super-Kamiokande, it will be possible to infer the neutral current component of the elastic scattering observed by Super-Kamiokande, and hence test the hypothesis that electron neutrinos oscillate into muon and tau neutrinos (rather than sterile neutrinos), even before SNO measures the neutral current rate directly [42].

Suppose that neutrino masses are confirmed - so what? I have heard it suggested that neutrino masses are just a trivial extension to the Standard

Model and hardly worth mentioning, and that learning the remaining MNS parameters is nothing more than stamp-collecting. It is true that one can simply add three light right-handed neutrinos (without Majorana masses) and couple them to the three left-handed neutrinos via Standard Model-like Yukawa couplings, to describe the neutrino spectrum. However a third family neutrino mass of 0.05 eV is much smaller than the tau lepton mass of 1777 MeV, and would require a tau neutrino Yukawa coupling 35 billion times smaller than the tau lepton Yukawa coupling, which looks rather unnatural when compared to the top-bottom quark mass ratio of about 40. On the other hand light right-handed neutrinos are not protected from receiving Majorana masses by any gauge symmetry, so it is perfectly natural for them to be much heavier than the weak scale, in which case naturally small neutrino masses can arise via the see-saw mechanism. In this case the scale of the heavy right-handed neutrino mass will be associated with new physics such as GUTs or string theory.

The origin of the heavy right-handed neutrino Majorana mass scale is in itself very interesting, and it may be useful to classify theories in terms of whether the right-handed neutrinos carry gauge quantum numbers at high energy (as in gauged $SU(2)_R$ models or $SO(10)$) or are gauge singlets even at high energy (as in $SU(5)$). For example if the right-handed neutrinos tranform as part of a gauged $SU(2)_R$ doublet, then anomaly cancellation predicts three right-handed neutrinos, and neutrino masses are inevitable. However if the "right-handed neutrinos" are really gauge singlets at high energies then the number and mass of such states is undetermined. Additional gauge singlets are also quite plausible in the gauged $SU(2)_R$ case. In general there may be several gauge singlets, with a sequence of vacuum expectation values and scales, making the see-saw mechansim quite a complicated problem, and this is typical in string theories. In all these cases some simplification might arise if one of the singlets plays the dominant role in the see-saw mechanism as in SRHND. A further window into high energy right-handed neutrino physics is provided by Leptogenesis [43]. The heavy right-handed neutrinos may be produced in the early universe, and decay out of equilibrium into higgs plus leptons, producing a lepton asymmetry as a result of CP violation in the lepton sector, which is then reprocessed by sphaleron interactions into the observed baryon number of the universe [43]. [3] The alternatives to the see-saw mechanism, large extra dimensions or R-parity violating supersymmetry, are hardly less exciting.

It is important to determine the detailed pattern of neutrino masses and mixing angles, since some day this will be related to the quark masses and mixing angles in the framework of an all-encompassing theory [5, 45]. To

[3]Note that the amount of CP violation in the lepton sector necessary to achieve leptogenesis does not imply that the Dirac phase δ will be large enough to be measurable at a Neutrino Factory, but it may be [44].

determine θ_{13}, the pattern of neutrino masses and the CP violating Dirac phase may require a Neutrino Factory. Without knowing the detailed spectrum, we shall most likely never know if we have the correct theory since we shall not know the low energy observables that we seek to explain. This is of course the normal way that science progresses, first the phenomena then the theory which lies underneath the observations. Eventually there is no doubt that knowing the neutrino masses and mixing angles will help to unlock the puzzle of all fermion masses and mixing angles, but we may have to wait for further direct information about new physics, as for example would be the case if superpartners are found at colliders, before all the pieces of the puzzle can be assembled. For example there is an interesting interplay between the slepton mass matrix and the see-saw parameters, in the framework of supersymmetry [46].

Neutrino oscillation physics is arguably the fastest developing area of particle physics at the moment. For example since 1998 more papers have appeared containing the word neutrino(s) than the word quark(s) in the title. With progress in this area showing no sign of slowing up as the new experiments successively come on line, this is a trend which seems likely to continue.

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